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What is paradoxical about the "Three-box paradox"?

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Abstract

This paper is a comment on quant-ph/0606067 by Ravon and Vaidman, in which they defend the position that the "three-box paradox" is indeed paradoxical.

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Some time ago, it was suggested ([1], [2]) that the description of a quantum system in the time interval between two measurements would involve aspects which are counter-intuitive and perhaps even paradoxical. One example which has been suggested involves a particle which might be in any of several boxes. This example has been given the suggestive title "the three-box paradox", but in order to not pre-judge the issue I will refer to this example as the "three-box story". Other authors ([3]–[7]) have criticized various aspects of the interpretations given by the authors of [1] and [2]. In particular, it has been argued that the three-box story should not be considered to be paradoxical, but in a recent article Ravon and Vaidman [8] have defended the claim that this story should indeed be called a paradox.

The three-box story concerns three boxes labeled A, B, and C, and a single spinless particle. Define $|A\rangle$ to be the state of the particle if it is in box A, with analogous definitions for the states $|B\rangle$ and $|C\rangle$. The particle is prepared in the initial state

$$|\Psi_i\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle + |C\rangle) \tag{1}$$

and post-selected to be in the state

$$|\Psi_f\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle - |C\rangle) \tag{2}$$

At some time after the preparation but before the post-selection an agent, let me call her Alice, looks into either box A or box B to see whether the particle is there; an elementary calculation shows that Alice surely finds the particle in whichever of these two boxes she happens to look.

Kirkpatrick [7] has described a purely-classical situation which parallels that in the three-box story. The obvious implication of this would be that since a classical story would not be paradoxical, the three-box story must also not be. Ravon and Vaidman dispute this implication by claiming that Kirkpatrick's story and the three-box story are not really analogous. Of course, since the three-box story includes quantum measurements, no classical story could reproduce it exactly; the question to be decided is: does Kirkpatrick's story capture those aspects of the three-box story which are asserted to be paradoxical?

So what is it about the three-box story which is asserted to be paradoxical? Here are two quotations from the paper by Ravon and Vaidman [8]; I will label these quotations RV1 and RV2. Near the bottom of page 1, Ravon and Vaidman write

RV1 The paradox in the Three-Box experiment is that at a particular time we can claim that a particle is in some sense both with certainty in one box, A, and with certainty in another box, B. Now, if a particle is certainly in A, then it is certainly not in B, and vice versa. Therefore, if a single particle is both certainly in A and certainly in B we have a paradox.

Then on page 2 they write

RV2 In the Three-Box experiment, the particle is *certain* to be found in A if searched for in A, and *certain* to be found in B if searched for in B instead.

Before agreeing that quotation RV2 does indeed describe a paradox, we should remember the condition under which it is supposed to be true, namely that the particle was post-selected in the state $|\Psi_f\rangle$ which was given in eq. 2. Let me say that the post-selection is accomplished by another agent (whom I will call Bob) measuring an observable (which I will call R) whose associated operator has as eigensubspaces the subspace generated by $|\Psi_f\rangle$ and the complement of that subspace. If when he measures R, Bob does indeed find the state $|\Psi_f\rangle$, I will say that the post-selection "succeeds". Then by including together with RV2 the explicit mention of the condition under which it is supposed to be valid, we get a statement I will label S1:

S1 If the post-selection succeeds, then

1. If the particle was searched for in box A, it was certainly found there, and
2. If the particle was searched for in box B, it was certainly found there.

Statement S1 has the same content as does quotation RV2 (with the condition which was implicit there made explicit), but Kirkpatrick could make an equivalent statement about his classical story (after correcting for the fact that his story involves a deck of cards rather than a particle and boxes [9]). Nevertheless, Ravon and Vaidman insist that the three-box story is paradoxical in a way in which Kirkpatrick's story is not; they define a "quantum paradox" to be "a phenomenon that *classical* physics cannot explain". Obviously that is a definition which no classical story can meet, but I would suggest that it does not represent what is usually understood by the term "paradox". Suppose that quantum physics permitted the single particle to suddenly become three particles, one in each box; that might not be explained by classical physics, but would not usually be called paradoxical. A "paradox" is usually considered to be something which (at least apparently) violates the laws of common sense, or perhaps even those of logic. And that is the way in which the advocates of the "three-box paradox" advertise it (at least when they are not arguing against Kirkpatrick's example); quotation RV1 clearly claims that the three-box story is in (at least apparent) conflict with laws of logic. Furthermore, any quantum measurement will have non-classical aspects, so if we were to accept the Ravon-Vaidman definition of quantum paradox, we might conclude that the three-box story does involve a paradox, but only to the extent that any quantum measurement does. I suspect that Ravon and Vaidman would want us to conclude more than that.

Kastner [6] has re-told the three-box story following the ordinary temporal order: First, the particle is prepared in the state $|\Psi_i\rangle$ given in eq. 1. Next, Alice looks into either box A or box B; either way she might find the particle (the probability that she finds it is 1/3) or she might not find it. Finally, Bob measures the observable R; in the cases in which Alice did find the particle in the box into which she looked, this post-selection might succeed (also with probability 1/3, as it happens), but in the cases in which Alice did not find the particle, the post-selection certainly does not succeed. Thus we can make a statement which I label S2:

S2
1. If the particle was searched for in box A and not found there, then the post-selection does not succeed, and
2. If the particle was searched for in box B and not found there, then the post-selection does not succeed.

According to statement S2, two different and incompatible scenarios (namely, particle searched for in box A but not found, and particle searched for in box B but not found) have a common consequence (namely, the post-selection does not succeed), but there is nothing at all strange about that. The calculation which leads to S2 does involve quantum interference, but quantum interference by itself is not usually considered paradoxical (at least, not now in the 21st century!) In fact, there does not seem to be any reason whatsoever to label as paradoxical the situation described by statement S2. But statements S1 and S2 are completely equivalent, so if S2 is not paradoxical, then neither is S1.

Quotation RV2 is essentially the same as statement S1, except that the restriction to cases in which the post-selection succeeds is implicit in RV2 (it is part of the definition of the three-box experiment) while in S1 that restriction is stated explicitly. Certainly Ravon and Vaidman never state, nor in any way imply, that RV2 would be valid without that restriction—if they had, their paper would simply be wrong, and that is not at all the case. Why then does RV2 at least give the impression of describing a paradox? What one judges to be paradoxical is to some extent a matter of personal psychology, but let me speculate that when the restriction to cases in which the post-selection succeeds is not made explicitly, the reader might tend to not consider the significance of that restriction. The reader might forget that there is never a situation in which the post-selection has already succeeded but for which Alice's looking into boxes A and B are both still possible. (Or, as pointed out by Bub and Brown [3], that the post-selected subensemble in which Alice looks into box A does not coincide with that in which she looks into box B.) In any event, when the three-box story is summarized as in statement S1, or even better, as in statement S2, there does not seem to be any reason to consider it paradoxical.

Statement S2 completely describes all of the relevant features of the three-box story, and shows that it is about as straightforward as any story involving quantum interference could be. Does this not indicate that any other account of this story which suggests that it might have paradoxical aspects must be misleading?

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[9] In parallel with statement S1, Kirkpatrick could say of his example "If the post-selection for Face = K succeeds, then 1. If Suit was checked to be S, it was certainly found to be so, and 2. If Suit was checked to be D, it was certainly found to be so."